Edge detection using compressed sensing

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Motivation for Edge detection
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Edges are an important feature of images for:

• Segmentation
Motivation for Edge detection

Edges are an important feature of images for:

- Segmentation
- Registration
Motivation for Edge detection

Edges are an important feature of images for:

- Segmentation
- Registration
- Restoration / Reconstruction
Motivation for Edge detection

Edges are an important feature of images for:

- Segmentation
- Registration
- Restoration / Reconstruction
- Target tracking
Example partial Fourier data
Example partial Fourier data
Example partial Fourier data
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Fourier transform

85% zero

Inverse Fourier transform
Example partial Fourier data

Fourier transform

Inverse Fourier transform

Aliasing and Gibbs

85% zero
Example partial Fourier data

Fourier transform

Aliasing and Gibbs

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Example partial Fourier data

Fourier transform

Inverse Fourier transform

Reconstruction using edges

Aliasing and Gibbs

85% zero
Outline

• Introduce data acquisition model
Outline

- Introduce data acquisition model
- Two approaches of detect edges from sampled data
• Introduce data acquisition model
• Two approaches of detect edges from sampled data
  1. Detect edges in a reconstruction
Introduce data acquisition model

Two approaches of detect edges from sampled data

1. Detect edges in a reconstruction

2. Detect edges directly from the given data without reconstruction
Forward Model

- Detect edges of $f$ from samples $g$

$$g = f * h + n$$
Forward Model

- Detect edges of $f$ from samples $g$

$$g = f * h + n$$

- $g$ is the measurement vector
- $*$ is the convolution operator
- $h$ is the point spread function
- $n$ is random noise
Example I: Partial Fourier data

- For example: MRI, tomographic reconstruction
- The Fourier transform of $h$ is an indicator function
Example II: Blurred data

- e.g. $h$ is a Gaussian point spread function
Approach 1: Use reconstructions
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- Two steps:
  1. Reconstruct signal
  2. Detect edges in reconstruction
Two steps:

1. Reconstruct signal
2. Detect edges in reconstruction

**Advantage:** Use classical reconstruction (e.g. Total Variation) and edge detection (e.g. Canny)
Approach 1: Use reconstructions

- Two steps:
  1. Reconstruct signal
  2. Detect edges in reconstruction

- **Advantage:** Use classical reconstruction (e.g. Total Variation) and edge detection (e.g. Canny)

- **Disadvantage:** Reconstruction may introduce artifacts
E.g. edges from TV reconstructions

Use total variation regularization

$$\min_{f} \{ \| f \ast h - g \|_2^2 + \lambda \| Lf \|_1^1 \}$$

$L$ is a first order approximation of the first derivative
E.g. edges from TV reconstructions

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Use total variation regularization

\[
\min_{f} \left\{ \| f \ast h - g \|_2^2 + \lambda \| Lf \|_1 \right\}
\]

\( L \) is a first order approximation of the first derivative.
E.g. edges from TV reconstructions

Use total variation regularization

$$\min_f \{ \| f \ast h - g \|_2^2 + \lambda \| L f \|_1 \}$$

$L$ is a first order approximation of the first derivative

False jumps due to “stair case” effect
Fix: Use higher order TV solutions

$$\min_f \left\{ \| f \ast h - g \|_2^2 + \lambda \| L^m f \|_1 \right\}$$
Fix: Use higher order TV solutions

\[
\min_{f} \left\{ \| f \ast h - g \|_2^2 + \lambda \| L^m f \|_1 \right\}
\]

- \( f \) has \( m^{th} \) derivative which is \textit{sparse}
  - e.g. \textit{Total Variation (m=1)}: 1\text{st} derivative sparse
Fix: Use higher order TV solutions

$$\min_f \left\{ \| f \ast h - g \|_2^2 + \lambda \| L^m f \|_1 \right\}$$

- $f$ has $m^{th}$ derivative which is *sparse*
  
  e.g. *Total Variation* ($m=1$): 1st derivative sparse

- $f$ is a *piecewise polynomial* of degree $m-1$
  
  e.g. *Total Variation* ($m=1$): piecewise constant
Fix: Use higher order TV solutions

\[
\min_f \left\{ \| f * h - g \|^2_2 + \lambda \| L^m f \|^1_1 \right\}
\]

- \( f \) has \( m \)th derivative which is \textit{sparse}
  - e.g. \textit{Total Variation} \((m=1)\): 1st derivative sparse

- \( f \) is a \textit{piecewise polynomial} of degree \( m-1 \)
  - e.g. \textit{Total Variation} \((m=1)\): piecewise constant

- \( L^m f \) is non-zero where polynomial segments meet (contact points)
  - e.g. \textit{Total Variation} between piecewise constant segments
Structure of higher order TV solutions

\[
\min_f \left\{ \| f \ast h - g \|_2^2 + \lambda \| L^m f \|_1 \right\}
\]

\[m = 1\]
Structure of higher order TV solutions

$$\min_{f} \left\{ \| f * h - g \|_2^2 + \lambda \| L^m f \|_1 \right\}$$

$m=1$

contact points, where constant segments meet
Structure of higher order TV solutions

$$\min_{f} \left\{ \| f \ast h - g \|_2^2 + \lambda \| L^m f \|_1 \right\}$$

$m=1$

contact points, where constant segments meet

$m=2$

contact points, where line segments meet
Structure of higher order TV solutions

$$\min_{f} \left\{ \| f * h - g \|_2^2 + \lambda \| L^m f \|_1 \right\}$$

- **m=1**: contact points, where constant segments meet
- **m=2**: contact points, where line segments meet
- **m=3**: contact points, where parabola segments meet
Structure of higher order TV solutions

\[
\min_f \left\{ \| f \ast h - g \|_2^2 + \lambda \| L^m f \|_1 \right\}
\]

- \(m=1\) contact points, where constant segments meet
- \(m=2\) contact points, where line segments meet
- \(m=3\) contact points, where parabola segments meet

\(m=2\) has no contact point at the jump location
Estimation of edges in a blurred signal

$m=1$

$m=3$
Estimation of edges in a blurred signal
Estimation of edges in a blurred signal

Algorithm

- Compute higher odd order TV restorations
- Find common contact points
Estimation of edges in a blurred signal

Algorithm

- Compute higher odd order TV restorations
- Find common contact points
50% Fourier coefficient example
Approach II: Directly use Fourier data
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• Assume $f(x)$ is piece-wise continuous and periodic
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- The conjugated Fourier sum converges to the jump function

$$S_N^\sigma[f] = i \sum_{k=-N}^{N} \text{sgn}(k) \hat{f}_k e^{ikx}$$
Approach II: Directly use Fourier data

- Assume $f(x)$ is piece-wise continuous and periodic
- Fourier coefficients are available
- The **conjugated Fourier** sum converges to the jump function

$$S_N^\sigma[f] = i \sum_{k=-N}^{N} \sigma\left(\frac{|k|}{N}\right) \text{sgn}(k) \hat{f}_k e^{ikx}$$

- $\sigma(\xi)$ is a so called **concentration factor** and speeds up the convergence (e.g Gelb, Tadmor 1999)
Concentration factor example
Concentration factor example

Polynomial concentration factor of order 1

Polynomial concentration factor of order 2
Concentration factor example

- Concentrates around edges but has unwanted oscillations
Apply method to any periodic function with only one unit jump at the origin $\Rightarrow$ oscillation pattern
Apply method to any periodic function with only one unit jump at the origin ⇒ oscillation pattern

Asymptotically for $N \rightarrow \infty$ : Matching waveform
The Matching Wave Form

- Apply method to any periodic function with only one unit jump at the origin $\Rightarrow$ oscillation pattern

- Asymptotically for $N \to \infty$ : Matching waveform

$$W_N^\sigma(x) = \frac{1}{N} \sum_{k=1}^{N} \sigma \left( \frac{k}{N} \right) \frac{\cos kx}{k}$$
Removing Oscillations
Removing Oscillations

- **Idea:** *deconvolve* matching wave
Removing Oscillations

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• deconvolution is ill-posed problem ⇒ need regularization
Removing Oscillations

- **Idea:** deconvolve matching wave
- Deconvolution is ill-posed problem ⇒ need regularization
- Jumps are sparse ⇒ Use $l_1$ minimization
Removing oscillations cont.

\[ y = \arg \min_u \| u \|_1 \quad \text{subject to} \quad \| W_N^\sigma * u - S_N^\sigma [f] \|_2^2 \leq \delta \]
Removing oscillations cont.

\[ y = \arg \min_u \| u \|_1 \quad \text{subject to} \quad \| W^\sigma_N * u - S^\sigma_N[f] \|_2^2 \leq \delta \]

- **Matching wave form** depends on the concentration factor \( \sigma \) but not on the function \( f \)
Removing oscillations cont.

\[ y = \arg \min_u \| u \|_1 \quad \text{subject to} \quad \| W_N^\sigma * u - S_N^\sigma[f] \|_2^2 \leq \delta \]

- **Matching wave form** depends on the concentration factor \( \sigma \) but not on the function \( f \)
- \( \delta > 0 \) in particular \( \delta \neq 0 \) because the **true wave form** depends on the function \( f \)
Removing oscillations cont.

\[ y = \arg \min_u \| u \|_1 \quad \text{subject to} \quad \| W_N^\sigma * u - S_N^\sigma[f] \|_2^2 \leq \delta \]

- Matching wave form depends on the concentration factor \( \sigma \) but not on the function \( f \)
- \( \delta > 0 \) in particular \( \delta \neq 0 \) because the true wave form depends on the function \( f \)
- Solve with any classical \( l_1 \) solver
Example 1

- 50% Fourier samples with different concentration factors

\begin{align*}
\text{i=1: polynomial CF p=1} & & \text{i=2: polynomial p=2} & & \text{i=3: exponential}
\end{align*}
Example 2

- 50% Fourier samples with 15% noise
Edge detection summary
Edge detection summary

- Two approaches:
Edge detection summary

- Two approaches:

1. Detect edges in total variation reconstructions
Edge detection summary

- Two approaches:
  1. Detect edges in total variation reconstructions
     - Compute odd higher order total variation reconstructions
Two approaches:

1. Detect edges in total variation reconstructions

   - Compute odd higher order total variation reconstructions
   - Find common contact points between polynomial segments
Edge detection summary

- Two approaches:
  
  I. Detect edges in total variation reconstructions
     - Compute odd higher order total variation reconstructions
     - Find common contact points between polynomial segments
  
  II. Detect edges directly from partial Fourier data
Edge detection summary

Two approaches:

I. Detect edges in total variation reconstructions
   - Compute odd higher order total variation reconstructions
   - Find common contact points between polynomial segments

II. Detect edges directly from partial Fourier data
   - Compute conjugated Fourier sum
Edge detection summary

- Two approaches:
  - I. Detect edges in total variation reconstructions
    - Compute odd higher order total variation reconstructions
    - Find common contact points between polynomial segments
  - II. Detect edges directly from partial Fourier data
    - Compute conjugated Fourier sum
    - Remove oscillations by a regularized deconvolution in one step
Conclusions
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- Two new methods to detect edges in
Conclusions

- Two new methods to detect edges in partial Fourier data
Conclusions

• Two new methods to detect edges in
• partial Fourier data
• data corrupted by blurring
Conclusions

- Two new methods to detect edges in
  - partial Fourier data
  - data corrupted by blurring
  - or both
Conclusions

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  - partial Fourier data
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- Combines edge detection and ideas from compressed sensing
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  - partial Fourier data
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- Can be extended to 2D
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- Two new methods to detect edges in
  - partial Fourier data
  - data corrupted by blurring
  - or both

- Combines edge detection and ideas from compressed sensing

- Can be extended to 2D

- Other approaches also combine compressed sensing and edge detection: for example (Tadmor&Zou, 2006), (Guo&Yin, 2010)
Extra: Lambda

false counts

false positives
missed jumps

p=1
p=3
exp

false counts

false counts

false counts